

Pearson Edexcel


## Tuesday 5 November 2019

Morning (Time: 1 hour 30 minutes) $\quad$ Paper Reference $\mathbf{1 M A 1 / 1 H}$

## Mathematics

## Paper 1 (Non-Calculator) Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser. Tracing paper may be used.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided - there may be more space than you need.
- You must show all your working.

- Diagrams are NOT accurately drawn, unless otherwise indicated.
- Calculators may not be used.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.
Write your answers in the spaces provided.
You must write down all the stages in your working.
1 Find the Lowest Common Multiple (LCM) of 108 and 120


LCM:(12) $\times(3) \times(3) \times(2) \times(5)=1080$
1080
(Total for Question 1 is $\mathbf{3}$ marks)

2 There are 60 people in a choir.
Half of the people in the choir are women.
The number of women in the choir is 3 times the number of men in the choir.
The rest of the people in the choir are children.
the number of children in the choir : the number of men in the choir $=n: 1$
Work out the value of $n$.
You must show how you get your answer.
$60 \div 2=30$ women in the choir
$r_{\text {half }}\left(\frac{1}{2}\right)$ are women.
$30 \div 3=10$ men in the choir
third ( $\frac{1}{3}$ ) of women = men
$60-10-30=20$ children in the choir
${ }^{\circ}$ rest are children
children: men $\div 10$

$$
\begin{gathered}
20: 10=2: 1 \\
(n: 1)
\end{gathered} \quad \text { so } n=2
$$

$$
n=\quad 2
$$

(Total for Question 2 is $\mathbf{4}$ marks)
3 Work out $1 \frac{3}{4} \times 1 \frac{1}{3}$
Give your answer as a mixed number.

$$
\begin{aligned}
& \eta=\frac{4}{4} \text { So } 1 \frac{3}{4}=\frac{4}{4}+\frac{3}{4}=\frac{4+3}{4}=\frac{7}{4} \\
& \eta=\frac{3}{3} \text { so } 1 \frac{1}{3}=\frac{3}{3}+\frac{1}{3}=\frac{3+1}{3}=\frac{4}{3} \\
& 1 \frac{3}{4} \times 1 \frac{1}{3}=\frac{7}{4} \times \frac{4}{3} \\
& =\frac{7 \times 4}{4 \times 3} \rightarrow 2 \times 12=24 \\
& =\frac{28}{12} \quad \text { so } \frac{28}{12}=2 \frac{4}{12} \text { or } \\
& \text { make top-heavy } \\
& \text { fractions. }
\end{aligned}
$$

(Total for Question 3 is 3 marks)

4 Use a ruler and compasses to construct the line from the point $P$ perpendicular to the line $C D$. You must show all construction lines.


1. Draw an arc from $P$ that intersects the line CD twice.
2. Set the compass to a shorter width and
draw an arc from $A$.
3. Draw an arc of the same width from B. Label the intersection $E$.
4. Connect points $P$ and $E$ with a straight line (use a ruler).

5 The diagram shows triangle $A B C$.

$A D B$ is a straight line.
the size of angle $D C B$ : the size of angle $A C D=2: 1$
Work out the size of angle $B D C$.

$$
\left.\begin{array}{rl}
75+51+2 x+x & =180 \\
126+3 x & =180 \\
3 x & =54 \\
18 & =x
\end{array}\right) \div \text { collect terms }
$$

$D C B=2 x=2(18)=36$
$B D C=180-36-51=93$
180 in a triangle

64 red bricks have a mean weight of 5 kg .
5 blue bricks have a mean weight of 9 kg .
1 green brick has a weight of 6 kg .
Donna says,
"The mean weight of the 10 bricks is less than 7 kg ."
Is Donna correct?
You must show how you get your answer.

$$
4 \times 5 \mathrm{Kg}=20 \mathrm{Kg}
$$

$$
\text { mean }=\frac{\text { total of weights }}{\text { number of bricks }}
$$

$5 \times 9 \mathrm{Kg}=45 \mathrm{Kg}$
mean $X$ number of bricks $=$ total of weights
$1 \times 6 \mathrm{Kg}=6 \mathrm{Kg}$

$$
\frac{20+45+6}{4+5+1}=\frac{71}{10}=7.1 \mathrm{~kg}
$$

$7.1 \mathrm{Kg}>7 \mathrm{Kg}$ so Donna is not correct.
(Total for Question 6 is $\mathbf{3}$ marks)

7 (a) Simplify $\left(p^{2}\right)^{5}$

$$
\left(p^{2}\right)^{5}=p^{2 \times 5}=p^{10}
$$

$$
\left(m^{a}\right)^{b}=m^{a \times b}
$$


(b) Simplify $12 x^{7} y^{3} \div 6 x^{3} y$

$$
\begin{aligned}
12 x^{7} y^{3} \div 6 x^{3} y= & \frac{12}{6} x^{7-3} y^{3-1}
\end{aligned}=2 x^{4} y^{2}, ~=m^{-b} \div m^{b}=m^{-b}
$$

$$
2 x^{4} y^{2}
$$

(Total for Question 7 is $\mathbf{3}$ marks)
$\qquad$

8 The accurate scale drawing shows the positions of port $P$ and a lighthouse $L$.


Scale: 1 cm represents 4 km .
Aleena sails her boat from port $P$ on a bearing of $070^{\circ}$
She sails for $1 \frac{1}{2}$ hours at an average speed of $12 \mathrm{~km} / \mathrm{h}$ to a port $Q$.
Find
(i) the distance, in km, of port $Q$ from lighthouse $L$,
(ii) the bearing of port $Q$ from lighthouse $L$.
i) speed $=$ distance - time so $\quad$ distance $=$ speed $X$ time
$=12 \times 1.5$
$=18 \mathrm{Km}$
$18 \mathrm{Km}-4 \mathrm{Km}=4.5 \mathrm{~cm}$ so $Q$ is 4.5 cm from $P$
Measure 070 from North with a protractor and mark Q 4.5 cm from P. Measure distance QL (about 5.25 cm ).
$5.25 \times 4=21 \mathrm{Km}$
ii) Using a protractor, measure angle clockwise from $L$ to $Q$. (about 325 )

| distance $Q L=$ | 21 |  |
| ---: | :---: | ---: |
| bearing of $Q$ from $L$ | $=$ | $325 \quad$km. |

(Total for Question 8 is 5 marks)

9 A car travels for 18 minutes at an average speed of $72 \mathrm{~km} / \mathrm{h}$.
(a) How far will the car travel in these 18 minutes?

$$
\text { distance }=\text { speed } X \text { time }
$$



18 minutes $=\frac{18}{60}$ hours

$$
\text { distance }=72 \times \frac{18}{60}=21.6 \mathrm{Km}
$$

## 21.6

 kmDavid says,
" 72 kilometres per hour is faster than 20 metres per second."
(b) Is David correct?

You must show how you get your answer.


No, David is incorrect because $72 \mathrm{Kmh}=20 \mathrm{~m} / \mathrm{s}$ (they are the same speed).

10 The cumulative frequency table shows information about the times, in minutes, taken by 40 people to complete a puzzle.

plot endpoints $\quad$\begin{tabular}{|c|c|}

\hline Time ( $\boldsymbol{m}$ minutes) \& | Cumulative |
| :---: |
| frequency | <br>

\hline $20<m \leqslant \underline{40}$ \& 5 <br>
\hline $20<m \leqslant \underline{60}$ \& 25 <br>
\hline $20<m \leqslant \underline{80}$ \& 35 <br>
\hline $20<m \leqslant \underline{100}$ \& 38 <br>
\hline $20<m \leqslant \underline{120}$ \& 40 <br>
\hline
\end{tabular}

(a) On the grid below, draw a cumulative frequency graph for this information.

(b) Use your graph to find an estimate for the interquartile range.

$$
\begin{aligned}
& 40 \doteq 4=10 \text { so find values at } 10 \text { and } 30 . \\
& \left.\begin{array}{l}
\text { Value at } 30=67 \\
\text { Value at } 10=46 \\
67-46=21 \text { so IQR }=21
\end{array}\right\} \text { read from graph }
\end{aligned}
$$

One of the 40 people is chosen at random.
(c) Use your graph to find an estimate for the probability that this person took between 50 minutes and 90 minutes to complete the puzzle.
> C.F at 50 minutes $=13$ C.F at 90 minutes $=37$
> $37-13=24$ people
> Probability $=\frac{24}{40}$

11 There are $p$ counters in a bag.
12 of the counters are yellow.
Shafiq takes at random 30 counters from the bag.
5 of these 30 counters are yellow.
Work out an estimate for the value of $p$.
Sample contains 30 counters.
Probability of yellow in sample: $\frac{5}{30}=\frac{1}{6}$

$$
\frac{12}{p}=\frac{1}{6} \rightarrow \begin{aligned}
& 6 \times 12=p \\
& p=72
\end{aligned}
$$


cross-multiply

72
(Total for Question 11 is 2 marks)
$12 T=\frac{q}{2}+5$
Here is Spencer's method to make $q$ the subject of the formula.

$$
\begin{aligned}
2 \times T & =q+5 \\
q & =2 T-5
\end{aligned}
$$

What mistake did Spencer make in the first line of his method?

5 should also be multiplied by 2.

13 (a) Write $\frac{5}{x+1}+\frac{2}{3 x}$ as a single fraction in its simplest form.

$$
=\frac{5 \times 3 x}{(x+1) \times(3 x)}+\frac{2 \times(x+1)}{(x+1) \times(3 x)} \leftarrow
$$

make a common denominator by multiplying top and bottom by the same value
$=\frac{15 x+2(x+1)}{3 x(x+1)} \leftarrow$ combine fractions and expand brackets

$$
\begin{equation*}
=\frac{15 x+2 x+2}{3 x^{2}+3 x}=\frac{17 x+2}{3 x^{2}+3 x} \quad \frac{17 x+2}{3 x^{2}+3 x} \tag{2}
\end{equation*}
$$

(b) Factorise $(x+y)^{2}+3(x+y) \leftarrow$ make the expression into two brackets

$$
(x+y)^{2}+3(x+y)=(x+y+3)(x+y)
$$

$$
\begin{equation*}
(x+y+3)(x+y) \tag{1}
\end{equation*}
$$

(Total for Question 13 is $\mathbf{3}$ marks)

14 The diagram shows a right-angled triangle.


All the measurements are in centimetres.
The area of the triangle is $27.5 \mathrm{~cm}^{2}$
Work out the length of the shortest side of the triangle.
You must show all your working.

$$
\begin{aligned}
& \text { area of a triangle }=\frac{\text { base } \times \text { height }}{2} \\
& 27.5=\frac{(x-2) \times(x+4)}{2} \\
& 27.5=\frac{x^{2}-2 x+4 x-8}{2} \\
& 27.5=x^{2}+2 x-8
\end{aligned} \quad \begin{aligned}
\text { collect } & \begin{aligned}
x & =-9: \\
\text { short side } & =(-9)-2 \\
& =-11
\end{aligned}
\end{aligned}
$$

length cannot be negative so discard this answer.

$$
\text { if } \begin{aligned}
& x=7: \\
& \text { short side }=7-2 \\
&=5
\end{aligned}
$$

$$
x=-9 \text { and } x=7
$$

15 Express $0.41 \dot{8}$ as a fraction.
You must show all your working.

$$
\begin{aligned}
x & =0.4188 \\
10 x & =4.18
\end{aligned}
$$

$1000 x=48.18$
find two multiples that cancel
$1000 x-10 x=418.18-4.18$ the recurring decimal out

$$
\therefore\left(990\binom{990 x=414}{x=\frac{414}{990}} \div 990\right.
$$

$$
\frac{414}{990}
$$

(Total for Question 15 is 3 marks)
16 (a) Rationalise the denominator of $\frac{22}{\sqrt{11}}$
Give your answer in its simplest form.

$$
\frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}=\frac{22 \sqrt{11}}{11}=\frac{22}{11} \times \sqrt{11}=2 \sqrt{11}
$$

equal to?

$$
\begin{equation*}
2 \sqrt{11} \tag{2}
\end{equation*}
$$

(b) Show that $\frac{\sqrt{3}}{2 \sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where $a$ and $b$ are integers.

$$
\begin{aligned}
\frac{\sqrt{3}}{2 \sqrt{3}-1} \times \frac{2 \sqrt{3}+1}{2 \sqrt{3}+1} & =\frac{\sqrt{3} \times(2 \sqrt{3}+1)}{(2 \sqrt{3}-1)(2 \sqrt{3}+1)} \\
& =\frac{(2 \times 3)+1 \sqrt{3}}{(4 \times 3)+2 \sqrt{3}-2 \sqrt{3}-1} \\
& =\frac{6+\sqrt{3}}{11}
\end{aligned}
$$

$17 \mathbf{A}$ and $\mathbf{B}$ are two similar cylindrical containers.

the surface area of container $\mathbf{A}$ : the surface area of container $\mathbf{B}=4: 9$
Tyler fills container $\mathbf{A}$ with water.
She then pours all the water into container B.
Tyler repeats this and stops when container $\mathbf{B}$ is full of water.
Work out the number of times that Tyler fills container $\mathbf{A}$ with water.
You must show all your working.
surface area $=\mathrm{cm}^{2}$
length $=\mathrm{cm}$
volume $=\mathrm{Cm}^{3}$
length $\quad 4: 9 \rightarrow \sqrt{4}: \sqrt{9} \rightarrow 2: 3$
volume $2: 3 \rightarrow 2^{3}: 3^{3} \rightarrow 8: 27$
$27 \div 8=3.375$
$=4$ (round to whole number)

18 The function f is given by

$$
\mathrm{f}(x)=2 x^{3}-4
$$

(a) Show that $f^{-1}(50)=3$

$$
\begin{aligned}
& +4\binom{x=2 y^{3}-4}{x+4=2 y^{3}}+4 \\
& x+4=2 y^{3} \\
& \frac{x+4}{2}=y^{3}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{aligned}
f^{-1}(50) & =\sqrt{\frac{50+4}{2}} \\
& =3 \sqrt{\frac{54}{2}} \\
f^{-1}(x)=3 \sqrt{\frac{x+4}{2}} & =3 \sqrt{27} \\
f^{-1}(50) & =3
\end{aligned}
\end{aligned}
$$

The functions g and h are given by

$$
\mathrm{g}(x)=x+2 \text { and } \mathrm{h}(x)=x^{2}
$$

(b) Find the values of $x$ for which

$$
\operatorname{hg}(x)=3 x^{2}+x-1
$$



$$
\begin{aligned}
h g(x) & =(x+2)^{2} \\
& =x^{2}+4 x+4
\end{aligned}
$$

$$
\begin{aligned}
& x^{2}\binom{3 x^{2}+x-1=x^{2}+4 x+4}{2 x^{2}+x-1=4 x+4}-x^{2} \\
& 2 x^{2}-3 x-5=0 \\
& (2 x-5)(x+1)=0 \leftarrow \text { factorise } \\
& x=2.5 \text { and } x=-1
\end{aligned}
$$

$$
\begin{equation*}
x=2.5 \text { and } x=-1 \tag{4}
\end{equation*}
$$

19 Given that $9^{-\frac{1}{2}}=27^{\frac{1}{4}} \div 3^{x+1}$
find the exact value of $x$.

$$
a^{\frac{1}{b}}=\sqrt{a} \text { and } a^{-b}=\frac{1}{a^{b}}
$$

$$
\begin{aligned}
9^{-\frac{1}{2}} & =\frac{1}{\sqrt{9}}=\frac{1}{3}=3^{-1} \quad\left(a^{b}\right)^{c}=a^{b \times c} \quad a^{b} \div a^{c}=a^{b-c} \\
27^{\frac{1}{4}} & =\left(3^{3}\right)^{\frac{1}{4}}=3^{\frac{3}{4}} \\
3^{-1} & =3^{\frac{3}{4}} \div 3^{x+1} \\
-1 & =\frac{3}{4}-(x+1) \\
\frac{7}{4} & =x+1 \\
\frac{3}{4} & =x
\end{aligned}
$$

## $\frac{3}{4}$

$x=$

20 The graph of $y=\mathrm{f}(x)$ is shown on the grid.

(a) On the grid, draw the graph with equation $y=\mathrm{f}(x+1)-3$
translate graph -3 units vertically and -1 units horizontally
Point $A(-2,1)$ lies on the graph of $y=\mathrm{f}(x)$.
When the graph of $y=\mathrm{f}(x)$ is transformed to the graph with equation $y=\mathrm{f}(-x)$, point $A$ is mapped to point $B$.
(b) Write down the coordinates of point $B$.

$$
y=f(-x) \text { is reflected in the } y \text {-axis. }
$$

The point A becomes point (2,1)

21 Sketch the graph of

$$
y=2 x^{2}-8 x-5
$$

showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

$$
\frac{-(-8) \pm \sqrt{\left.(-8)^{2}-4(4)-5\right)}}{(2(2)}=\frac{8 \pm \sqrt{104}}{4}=2 \pm \sqrt{\frac{13}{3}}
$$

$$
2 x-8 x-5=2(x-2)^{2}-13
$$

when $x=0, y=-5$ turing point at $(2,-13)$ $y$-intercept at ( $0,-5$ )
put into completed square form

(Total for Question 21 is 5 marks)
$22 A, B, C$ and $D$ are four points on a circle.

$A E C$ and $D E B$ are straight lines.
Triangle $A E D$ is an equilateral triangle.
Prove that triangle $A B C$ is congruent to triangle $D C B$.
use ASA (two angles and the side between them) to prove congruency
$\triangle A E D$ is equilateral so angles $A \hat{E D}, A D E$ and $E \hat{A} D=60^{\circ}$
CEB is also equilateral because triangles in opposite segments are congruent, so $C \hat{E} B, C \hat{C B E}$ and $E \hat{C} B=60^{\circ}$
ANGLES $A \hat{C B}$ and D $\overline{B C}$ are equal ( $60^{\circ}$ )
SIDE BC is common to both triangles
$\widehat{D C A}=\hat{D B A}=4$ because angles in the same segment are equal
ANGLE $\hat{D C B}=\hat{A B C}=4+60^{\circ}$

Therefore, triangles ABC and DCB are congruent by ASA
(Total for Question 22 is 4 marks)

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